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ABSTRACT

There are selected philosophies in the teaching of mathematics which can provide guidance to the teacher in developing the curriculum and also a framework for teaching and learning. This paper discusses four such philosophies of teaching mathematics: Idealism, Realism, Experimentalism, and Existentialism. Idealism stresses that students live in an idea-centered mathematical world with mental development of the student as the primary goal of instruction. The mathematics curriculum is viewed here as part of the general education curriculum with abstract content prized higher than concrete or semi-concrete content. Realism emphasizes that a person can know the real world in whole or in part as it really is, and students can become more knowledgeable of the real world by attaining precise, measurable, stated objectives in mathematics. Experimentalism emphasizes students learning, that which is useful and utilitarian and also emphasizes problem solving and cooperative learning. Existentialism stresses individual choices made by a student in selecting sequential tasks and experiences in mathematics. It is concluded that the teacher needs to select to implement that philosophy which assists a student in attaining optimally. The use of diverse philosophies of education to provide for individual differences should assist each student in learning as much mathematics as possible. (JRH)

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Philosophy of Teaching Mathematics

by Marlow Ediger

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PHILOSOPHY OF TEACHING MATHEMATICS

There are selected philosophies in the teaching of mathematics which can provide guidance to the teacher in developing the curriculum. Each teacher has selected concepts and generalizations which provide a framework for teaching and learning. A study of the philosophy of education may develop a reservoir from which the teacher may secure the background knowledge, attitudes, and skills to do a quality job of teaching learners. Ozman and Craver (1990) wrote:

> A study of philosophy of education seems imperative today, for we are in a critical era of transition. There has always been change, but seldom at our present accelerated rate, creating in many individuals what Alvin Toeffler has called the sickness of "future shock." In such an age, it is easy for people either to embrace more and more with little thought to eventual consequences or to resist change with little or no matter what. Educational philosophers, regardless of the particular theory they embrace, suggest that the solutions to our problems can best be achieved through critical and reflective thought. In one sense we can say that philosophy of education is the application of philosophical ideas to educational problems. We can also say with equal force that the practice of education leads to a refinement of philosophical ideas. From this viewpoint, educational philosophy is not only a way of looking at ideas but also learning how to use them in the best way. No intelligent philosophy of education is involved when educators do things simply because they were done in the past. A philosophy of education becomes significant at the point where educators recognize the need to think clearly about what they are doing and to see what they are doing in the larger context of individual and social development.

Idealism in Teaching Mathematics

Idealism is one of the oldest philosophies available which may assist the mathematics teacher to select objectives, learning opportunities, and evaluation procedures for pupils. Plato (427- 347 BC) advocated idealism as a philosophy of education in ancient Athens. Above Plato's academy door, it stated that "no one is to enter unless they know mathematics." He believed mind to be superior to the body.

Thus a strong academic curriculum in mathematics should be in the offing. Upon death, the mind/soul survives whereas the body decays. The mind must rule the body so that higher levels of choices are in evidence. It is the body that brings an individual to lower or inferior levels of choices and decisions. A study of mathematics assists the learner to attain well mentally. In the Forms (heaven), perfection is there in that a perfect something exists, such as different number systems, and geometrical figures, plain and solid. The here and the now on the changing earth is inferior to what is in the unchanging Forms. Thus a triangle, square, parallelogram, and circle, for example, in the here and now are imperfect models of what is perfect in the Forms. The same is true of all things and life on earth. What exists in the stable Forms is much superior to the world of change here on earth.

One only receives ideas about the Forms according to Plato. A person cannot perceive the Forms as they truly are, but receives ideas through thought, mind, meditation, and intellectual endeavors. An idea centered mathematics curriculum pertaining to the abstract assists in achieving thinking individuals who reflect upon subject matter acquired. The well educated and the abstract thinkers have abilities to perceive or receive ideas pertaining to The Forms. Wisdom is a necessary prerequisite to perceive The Forms.

Idealism as a philosophy of education still receives much attention today. A mathematics teacher who is an idealist tends to emphasize mental endeavors as being superior to the physical and its emphasis. The mind is what is truly real about the person. Thus the mathematics teacher needs to stress pupils attaining abstract content in mathematics since this will aid mental development. Higher cognitive level objectives need to be selected and implemented in the mathematics curriculum. These objectives pertain to pupils being able to think critically, synthesize content, and appraise what has been acquired. Mind is real and needs to be developed, according to idealism as a philosophy of teaching mathematics.

Concrete learning opportunities consisting of the use of real

objects, and the semiconcrete emphasizing use of illustrations, should be stressed only if they guide learners to understand abstract ideas in mathematics. The focal point of instruction is ideas and mental development. One receives ideas of the natural and social environment only, not a replica of the real world. All information is developed by the mind. Ideas are then secured about the natural and social facets of life. What is in back of this world is mind and the spiritual, not the physical. Scope and sequence in mathematics emphasizes mental and intellectual development of the pupil pertaining to the following topics:

1. base ten system of numeration, estimating, as well as understanding positive and negative integers.
2. addition, subtraction, multiplication, and division on whole numbers, the integers, rational numbers, and irrational numbers..
3. geometry with its space figures, including plane (squares, rectangles, triangles, parallelograms, and circles, among others) and solid (spheres, cylinders, cones, prisms, and Platonic solids). 4. common and decimal fractions, as well as per cents.
5. measurement including linear, square, and cubic.
6. graphs, tables, statistics and probability.

For the above named topics, pupils with teacher guidance need to study each in depth with emphasis placed upon learners attaching meaning to content being taught. Mathematics as general education is salient in developing mental maturity to work with numerals and number in the abstract. Critical thinking in the mathematics curriculum stresses mental development. Reason and intelligence are necessary to achieve fully in mathematics. The rational being then becomes increasingly mature mentally to use intelligence in dealing with the world of number and numerals. The teacher stimulates pupils to achieve using a variety of learning opportunities emphasizing inductive and deductive methods of thinking. Idealists stress the concept of purpose for each human being in a purposive world. There is purpose involved in learning mathematics. Th purpose involves, among other things, the

development of the spiritual facet of the person. Human beings are not a part of the animal world, according to idealists. Rather they transcend that level and are endowed with rational powers that animals do not possess. Mathematics as an academic discipline can assist pupils to reach out from the finite toward the Infinite, in achieving intellectual and rational goals. Mathematical truths are a priori and thus have always existed. For example, any basic number sentence in mathematics such as 12 x 10 =120 has always been true, prior to human experience. Each pupil must be guided by a competent and academically inclined teacher in discovering preexistent truths in mathematics.

For the idealist, mathematics presents content to pupils to encourage the development of reasoning persons in which the mind achieves in the direction of the Infinite, the unlimited in terms of attaining an ideal. Mind, not matter, represents ultimate reality. The mind and mathematics content stress reaching toward the Ideal or Infinite in achieving a priori content.

Since idealists in mathematics tend to recommend mental development of the learner as a major goal of instruction, a quality series of mathematics textbooks might well provide appropriate scope and sequence in the curriculum. The teacher's role here is to assist pupils to attain optimally in thinking mathematically. Brubacher (1966) wrote:

> The most prolific writer on the idealist philosophy of education in the twentieth century was Herman Harrell Horne (1874-1946). At a time when idealism was fast fading as the dominant American theory of education, Horne managed to draw together the various strains of idealism into their more systematic educational exposition. In addition to much that is already familiar, he made two points of his own. One is his emphasis upon volition and effort in learning. The pupil is like the plant, he agreed with Froebel, in that his response is self active. But the child is unlike a plant . Horne continued, in that he can withhold his response. Hence the ultimate responsibility for getting an education rests on the will of the pupil. All education therefore is self education: it is the voluntary effort put forth by a self active mind. If effort is aided and abetted by interest, well and good. If not , then like Kant, Horne urged that the pupil in any case put forth effort in obedience to what he ought to do.

A second and more notable point in Horne's exposition is the fact that he did not make any significant alteration in the developmental theory of education in the light of the Darwinian theory of evolution, which was introduced to the world after the deaths of Hegel and Froebel. To be sure, Horne saw that evolution had made the developmental process irreversible and unrepealable, in contrast to the Aristotelian pattern of matter endlessly reproducing the cycle of changes demanded by its form. The Absolute, however, had no difficulty in assimilating this new theory of development, for Horne could still say that the Absolute is; only the finite becomes. Pedagogically speaking , this seems to mean that through education the child still becomes in time what he was meant eternally to be.

Horne was a strong advocate of a subject centered curriculum. An idea centered curriculum in mathematics is then in evidence. The abstract numbers and numerals, the symbols of operation on numbers, as well as the different formulas in determining area and volume, among others, might well provide a significant set of lessons and units in mathematics. Concrete and semiconcrete materials may be used to guide pupils to achieve well in the abstract.

Realism and the Mathematics Curriculum

The mathematics teacher who stresses realism as a philosophy of education believes in using the methods of science in teaching and learning situations. Objective evidence, irrespective of the subjective person, is inherent in mathematics. Thus, subject matter in mathematics is true independent of the observer or person. Precision is a key word to use in teaching mathematics, according to the realist. A realist likes accurate descriptions of what exists. For example, he/she does not care for a person saying that the temperature reading in a room is comfortable. Rather, the exact temperature reading is wanted such as 22 degrees Celsius. If a person states that his/her blood pressure reading is normal, the realist desires to know the precise blood pressure reading using numerals for the systolic and diastolic readings. A teacher who emphasizes that his/her pupils are attaining well does not satisfy the realist critic. Rather numerical results are wanted to ascertain

how well learners are attaining, such as grade equivalents, percentile ranks, quartile deviations, as well as standard deviations from the mean and other derived or standard scores. Testing pupils to notice achievement is quite typical of the philosophy of realism. Thus standardized norm referenced tests may be used to gather data on learner progress. Formative and summative tests are recommended to be given to learners to notice pupil progress in mathematics. The former is given to learners within an ongoing unit of study to monitor achievement along the way, as well as make needed changes in teaching. The summative test is given at the end of a unit of study in mathematics so that changes may be made, if evidence warrants, the next time the same unit is taught.

A mathematics teacher then who is a realist desires objectives of instruction to be stated in measurable terms, prior to instruction. The following are examples:

1. The pupil will add correctly ten number pairs, each containing single digit addends.

2. Given four geometrical figures, the learner will accurately compute the area of each.

3. The learner will change five common fractions to decimals and then to per cents.

4. Given three dimensional values for a rectangular prism, a triangular solid, a cylinder, and a pyramid, the pupil will compute accurately the volume of each geometrical solid.

5. Given data pertaining to the federal budget, the pupil will construct a line graph, a bar graph, and a picture graph.

For each of the above named objectives, pupils will reveal as a result of instruction if they have been successful in goal attainment. These objectives are stated with precision so that the mathematics teacher knows exactly what is to be taught. There is no guesswork in terms of what pupils are to learn. There are realists who advocate that the teacher announce prior to instruction what pupils are to learn as

stated in the objective(s). Pupils then do not need to out guess the teacher in terms of what is expected of them as learners. When ascertaining how much pupils have learned as a result of instruction, the teacher receives numerical results such as the per cent of correct responses from a test of each pupil. The results from each pupil could also be computed to secure percentile ranks. Derived scores based on the normal distribution curve would indicate the number of standard deviations above and below the mean for each pupil.

A mathematics teacher who is a realist in terms of philosophy of education desires precise objectives for learner attainment. He/she matches the learning opportunities with the stated specific objectives so that an increased number of objectives will be achieved by pupils. What is in the learning opportunities then harmonizes with what is stated in the objectives, no more and no less. Appraisal procedures harmonize with the objectives of instruction. Validity in appraisal is then in evidence. Results from the appraisal determine the number of objectives achieved satisfactorily by the learner. Results for the appraisal are objective in that independent of any evaluator, the number of correct responses would be the same each time. Subjectivity is then eliminated in the appraisal process.

Pertaining to realism as a philosophy of education, Bowyer (1970) write the following:

We have noted that there are different forms of naturalism and of idealism. The same is true of realism, which makes it difficult to pinpoint the distinguishing features of realism and to define the realist point of view. One element that the various forms of realism do have in common is a rejection of the idealist theory of knowledge that the various qualities of experience depend upon knower for their existence. Realists believe that the universe is composed of real entities that exist in themselves. These entities can be known, and their existence is not dependent upon a knower or perceive. Although realists can argue on this point, they do not all agree when they attempt to build a metaphysical system. Here their views range from pluralism to dualism to monism.

The realist's epistemological views include epistemological monism where it is held that objects are presented in consciousness, and epistemological dualism where objects are thought to be

represented. The monists define mind as a relation between the organism and an object, while the dualist identify the mind more closely with the organisms. Realists do have a common tendency to view the world as the mechanism described by the physical scientists , and they generally believe in determinism, in orderliness in the universe, and in the objectivity of nature. The unifying theory of realism is that knowledge is thought to have a universal character and comes to man through his sensory capacity. The realists have confidence in their assertions about reality and value which is most discerning to pragmatists.

Since mathematics and its component parts are independent of any person, that is the content is objective and not subjective, precision and complete accuracy of answers to questions and problems are possible. Mathematics probably possesses the most objective subject matter as compared to other academic disciplines. This makes realism as a philosophy of education very useful in choosing precise objectives for pupils to achieve. The learning activities might well be aligned with the objectives and the appraisal procedures may be used to assess learner performance against the stated objectives.

Experimentalism and the Mathematics Curriculum

The world of experience represents ultimate reality for the experimentalist. The realist believes that one can know the real world as it truly is in whole or in part. Also the real world exists independent of any observer or human being. The idealist believes that one can only know ideas about the real world, not as it is truly is.

With knowing what is experienced only, the experimentalist realizes that change is all around us. Our perceptions change in time and place. Life in society continually changes. Thus problems arise which need identification. Each problem is life- like and reality based, not fictional. Clarity in problem selection is relevant. Vague, hazy problems do not lend themselves to solutions. An hypothesis is developed for the identified problem. The hypothesis is actually an educated guess or answer to the chosen problem. The hypothesis is not absolute, but tentative. The hypothesis is then subject to testing in a

life-like situation. The consequences of the testing reveal the correctness or the lack thereof pertaining to the stated hypothesis. It is easy to understand how experimentalism with its problem solving situations is very relevant in ongoing lessons and units of study in mathematics. Problem solving is at the heart of the mathematics curriculum.

Problems should come from pupils in the world of society. Utilitarian problems are then identified, not textbook story problems. A practical mathematics curriculum is then in evidence. What is useful in the mathematics curriculum is desired in terms of objectives, learning opportunities, and evaluation procedures. The everyday experiences of people in society pertaining to mathematics provides content then for the experimentalist curriculum. Within a mathematics unit being studied, the learners choose problems to solve. Ediger (1994- 1995) wrote the following:

Problem solving requires deliberation in finding solutions to the unknown. There have been many approaches emphasized as problem solving. One approach has been to use word or story problems from the basal textbook. Higher levels of cognition can be emphasized here depending on the quality of the problem in the textbook. However, the problems therein may not be accepted by learners as having purpose. They may have outdated information in the problem which does not harmonize with what the real world of society has to offer today. Students can then not relate personally to these word or story problems as having merit.

A second approach at stressing problem solving has been for the teacher to devise the problems for students to solve using worksheets. Here again, the problems may emphasize higher levels of cognition, but they fail to arouse student interest and meaning. For problem solving to accrue, students must be involved in working on the life like which occurs in society. In the societal arena, there are problems in mathematics which need solving. These problems have personal meaning and purpose for the learner. Thus, learners individually or within a committee identify a problem. The identified problem must be specific enough so that it can be solved, but not so specific that rote learning is involved to recall answers. There is perplexity involved in the selected problem in that the student has to think deeply of how to obtain a necessary solution. Next, the learner develops a hypothesis or answer to the problem. The hypothesis is tentative, not an absolute.

The hypothesis is an educated guess, based upon the best information that the student has. Each hypothesis, being tentative, needs testing. The testing is done situationally and not on a paper- pencil test. If the consequences re positive, the hypothesis stays as is. Should it be warranted, the hypothesis is revised. New problems may be chosen as the tentative answers are chosen or as the hypothesis is developed. Problems might arise as any hypothesis is revised. Problem solving in mathematics is open-ended, not factual nor recall of isolated bits of information. Problem solving does not emphasize memorizing of content. Rather content is acquired to gather data to solve problems.

...which provide experiences for learners in the mathematics curriculum. These are the following, provided as examples:

1. planning how a pupil is to spend his/her weekly money allowance.

2. planning objectives, learning opportunities, and evaluation procedures with pupils for sequential lessons and units in mathematics which stress practical experiences.

3. planning an experimentalist mathematics curriculum which harmonizes with the National Council Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards for School Mathematics (published by NCTM, 1906 Association Drive, Reston, Virginia 22091- 1593).

4. planning how to divide cookies among a certain number of children who are involved in the lesson presentation. When teaching mathematics, there are numerous situations such as these, whereby pupils need to be actively involved in decision- making.

5. planning a class party related to a holiday in which mathematics is heavily used such as how many cookies, cup cakes, and soft drinks to purchase.

Mathematics teachers need to be creative in thinking about developing and implementing an experimentalist curriculum. There are numerous experiences which can be included in mathematics lessons and units of study emphasizing the practical and the utilitarian in life-like problem solving situations. Atkinson and Maleska (1965) wrote the following:

To a follower of Dewey, education has two sides -- psychological and social; neither may be subordinated or neglected. The psychological nature of a child forms the basis for his education -- it is the teacher's responsibility to make full use of his natural, spontaneous activities. Describing the original nature as being spontaneously impulsive rather than passive, Dewey divided impulses into four kinds; the societal impulses , of communication or conversation; the constructive impulse to make things; the impulse to investigate things; and the impulses of artistic or creative expression.

With these impulses in mind, said Dewey, the school must be changed from a place for sedentary listening to one for active doing or working. The teaching processes must be planned to allow the child to learn wherever possible by his own experiences and, in that way, to acquire the habit of thinking. A proper solution to any problem demands intelligent thinking which becomes the principle factor in the ability to cope with new situations. Thinking as Dewey defined it is the use of the meanings of past experiences in interpretations of new situations.

Dewey felt that when the psychological and the social approaches to learning are separated, there is produced either a forced and external education in which freedom of the individual is subordinated to a preconceived notion of what society should be, or else a barren and formal development of the mental powers in which the learner has little idea of the use to be made of what is being learned. The school is primarily a social institution because its processes are basically no different from those going on continuously in life outside the classroom.

Therefore, Dewey claimed, the manner in which pre-school learning has been taking place should suggest to a teacher the physical and mental growth. The school ideally should be that form of social life into which can be concentrated those factors that most effectively cause a child to share the accumulated knowledge and skills of the race. Education can be considered as proceeding most satisfactorily whenever the individual is actively participating in social relationships with others.

Existentialism and the Mathematics Curriculum

Existentialists stress the individual choosing and making decisions. To be sure, it is very salient that each pupil learn to engage in the making of choices. Life consists of making choices. Experimentalism emphasized also that pupils choose and make decisions, but usually within a committee setting. The belief exists in experimentalism that a pupil is a member of society presently and should be actively involved in the mathematics curriculum, but within a committee setting.

Existentialism emphasizes the individual as one who should determine his/ her curriculum within a flexible framework. The teacher assists the pupil in achieving the latters goals. I will mention a few other tenets of existentialism which may or may not apply to the mathematics curriculum. One first exists and then determines his/ her essence. Thus the individual pupil should be heavily involved in determining goals, learning opportunities, and evaluation procedures in mathematics. I truly believe this to be a difficult method of teaching, but it certainly has its values and benefits. In all of teaching, it is the learner that is the focal point of instruction. Jean Jacques Rousseau (1712- 1776) in his book Emile (see Brubacher, 1966) emphasized a one on one relationship between teacher and pupil. Thus a pupil would be taught by a mentor or teacher. Here, the teacher could truly provide for individual differences (one pupil and one teacher). The learner asks questions that would be of personal interest. The out of doors or nature provides the necessary curriculum for the pupil, according to Rousseau. The teacher then assists the pupil to find the needed information. Induction as a method of teaching is used here. The pupil does not need to depend upon other pupils for help in learning, but is to be an independent being, removed from the ills of society. Nor is the learner hindered in optimal achievement since no other pupil is there to hold the former back. The pupil does not need to gauge his (a boy in this case) learning against that of others in making comparisons. Uncomfortable comparisons between learners in achievement then can not be made in the one on one teaching situation. Rousseau's philosophy of instruction had definite tenets of existentialism. Which are selected mathematics experiences for pupils that Rousseau recommenced?

1. estimating the height of a cherry tree so that an appropriate ladder may be found or made to reach and pick cherries.
2. measuring the size of boards to make necessary items and objects.
3. becoming independent as a carpenter so that one does not need

to be a servant of others. (Rousseau was very critical of norms in society). In being a carpenter, arithmetic and geometry are salient to learn within the framework of life like situations.

Rugged individualism can be a term used to describe existentialism. Soren Kierkegarrd (1813- 1855), a theistic existentialist, advocated the the person is first born and then finds his/her essence, meaning the individual must find his/her own purposes in life. These purposes or goals are not given to anyone, but must be found. The individual makes the self in an open ended universe, very limited in restrictions. Prior to this time, most philosophers stressed that the essences or purposes of persons were given to all first, and then the individual would be more certain as to what his/her role in life would be. Idealism was a prominent philosophy during the centuries and emphasized that the Infinite was ultimate reality and had purposes established for all. Kierkegarrd was a theistic existentialist who also believed that the Absolute was ultimate reality; however a long struggle was necessary in reaching this goal involving personal choices made. Jean Paul Sartre (1905- 1980) emphasized atheistic existentialism as a philosophy of life. He also emphasized, as did Kierkegarrd, that an individual is born and then must find his/her own essence or purposes. With no Absolute, Sartre stressed that there is no one to manipulate the individual from above to determine purposes in life. Sartre's famous words that "Man is condemned to be free" certainly would make for a world of free choices for the individual. There are no absolutes.

Sartre (1971) in his essay "Man is Freedom," wrote the following:

It is strange that philosophers have been able to argue endlessly about determinism and free will, to cite examples in favor of one or the other thesis without ever attempting first to make explicit the structure contained in the very idea of action. The concept of an act contains, in fact, numerous subordinate notions which we shall have to organize and arrange in a hierarchy; it is to produce an organized instrumental complex such that by a series of concatenations and connections the modification effected on one end of the links causes modifications throughout the whole series and finally produces an anticipated result. but this is not what is important for us here. We should observe first that an action is on principle intentional. The careless smoker who has

through negligence caused the explosion of a powder magazine has not acted. On the other hand the worker who is charged with dynamiting a quarry and who obeys the given orders has acted when he has produced the expected explosion; he knew what he was doing or, if you prefer, he intentionally realized a conscious product.

This does not mean, of course, that one must foresee all the consequences of his act. The emperor Constantine, when he established himself at Byzantium, did not foresee that he would create a center of Greek culture and language, the appearance of which would ultimately provoke a schism in the Christian church and which would contribute to the weakening of the Roman Empire. Yet he performed an act just in so far as he realized his project of creating a new residence for emperors in the Orient. Equating the result with the intention is here sufficient for us to be able to speak of action. But if this is the case, we establish that the action necessarily implies as its condition the recognition of a "desideratum;" that is, of an objective lack or again a negatite...

Stumpf (1971) wrote:

Whether they were theists or atheists, the existentialists all agreed that traditional philosophy was too academic and too remote from life to have any meaning for them. They rejected systematic and schematic thought in favor of a more spontaneous mode of expression in order to capture the authentic concerns of concrete existing individuals. Although there is no "system" of existentialist philosophy, its basic themes can, nevertheless, be discovered in some representative existentialist thinkers.

Existentialists believe strongly in conscious choices made by individuals as being desirable. Moral judgments made in an atmosphere of freedom is a key concept stressed by existentialists.

An existentialist mathematics teacher needs to give learners as many options as possible in learning. The pupil chooses that option in a very open ended mathematics curriculum. Most teachers of mathematics would tend to feel that existentialist philosophy is too free of borders and boundaries. Mathematics has its own scope and sequence. The scope and sequence has much agreement in and among mathematics educators. I would like to describe a mathematics unit which a few of my student and regular teachers have used. The approach I will describe emphasizes the use of learning stations. The mathematics teacher here

needs to decide upon the number of stations needed. Perhaps for twenty- five pupils, there should be at least eight stations. Each station must possess concrete semiconcrete, and abstract materials of instruction. Also at each station, there is a task card which lists possibilities for pupils individually to choose from in terms of learning activities. The pupil may select which station and which tasks to work on sequentially. There should be an adequate number of tasks so that a pupil may omit that which does not possess perceived purpose. Sequence resides within the learner, not textbooks nor the teacher, in that the pupil orders his/her own experiences. If the pupil cannot find a station or task which meets personal purposes, he/she may plan with the teacher which learning opportunities to complete in mathematics. The learner is the chooser in deciding upon these tasks. The teacher encourages, assists, and guides the pupil in finding tasks and materials to complete that which has perceived purpose. Tasks at the different stations should have individual endeavors as well as those which stress committe work. The pupil then can work individually or with others, depending upon perceived purpose. In all cases, pupils individually sequence their very own learning opportunities.

There are mathematics teachers who stress additional tenets of existentialism in their teaching. The following are examples:

1. having pupils choose extra work to do in mathematics, beyond that which is required.

2. completing a contract with individual learners to indicate what he/she is to complete. The contract lists specifically what a pupil wishes to complete with a due date listed. What is in the mathematics contract represents that which the learner desires to complete with teacher assistance, not teacher direction.

3. using teacher- pupil planning in the mathematics curriculum in which the latter determines what will be learned in sequence with instructor guidance. Thus the objectives, learning opportunities, and evaluation techniques are chosen by pupils with teacher guidance in mathematics.

Each of the above three enumerated items contains mathematics learning opportunities which can be incorporated into any classroom. To stress tenets of existentialism, the teacher must lean upon pupils in determining what they wish to learn. From within or intrinsically, the learner is the decision maker in terms of selecting objectives, learning opportunities, and evaluation procedures in the mathematics curriculum.

Pertaining to existentialism as a philosophy of education and humanism as a psychology, Ediger (1994- 1995) wrote:

Humanism, as a school of thought in psychology, emphasized input form students into the mathematics curriculum. Input for students may stress problem solving or it may stress other kinds of experiences. A learning centers approach may be sued. The teacher develop the different centers with approximately five tasks per center. Teacher- pupil planning may be in evidence to plan the centers and the tasks. There needs to be more tasks at the different centers than what any student can complete so that decision- making is possible in choosing what to learn and what to omit. Time of task is important!

The student decides what to learn sequentially. A psychological mathematics is then in evidence when the learner chooses what to learn and what to omit. Sequence resides within the student, not the teacher nor in textbooks.... Tasks which choose, from among others, need to possess challenge, interest, and purpose. The mundane and routine should not be a part of the learning activities offered. If tasks do not meet personal needs of learners, student- teacher planning can be implemented so that the former might work on activities that do motivate. The individual then chooses which tasks to pursue and which to omit, be they problem solving or other kinds of experiences. There is no core or body of knowledge which all should pursue and complete.

In Closing

Four philosophies of teaching mathematics were discussed. Idealism stressed that pupils live in an idea centered mathematical world, but not an objective real world. Mental development of the pupil is a number one goal of instruction. The mathematics curriculum is viewed here as a part of the general education curriculum. Abstract content is prized higher that that which is concrete and semiconcrete.

Ideas only can be known by an idealist. One only receives ideas of the real world of the realist. Ideas alone are also received of the experiences that experimentalists say can be known only. Realism emphasized that a person can know the real world in whole or in part as it really is. With pupils attaining precise measurable stated objectives in mathematics, they become more and more knowledgeable of the real world as it truly is. Each objective attained assists the learner in knowing more and more about the real world as it truly is, not merely ideas of this world. Bertrand Russell (Quoted in Wahlquist 1942) wrote:

> The first characteristic of the new philosophy is that it abandons the claim to a special philosophic method or a particular brand of knowledge to be obtained by its means. It regards philosophy as essentially one science, differing from the special sciences merely by the generality of its problems, and by the fact that it is concerned with the formation of hypothesis where empirical evidence is still lacking. It conceives that all knowledge is scientific knowledge, to be ascertained and proved by the methods of science. It does not aim, as previous philosophy has usually done, at statements about the universe as a whole, nor at the construction of a comprehensive system. It aims only at clarifying the fundamental ideas of the sciences, and synthesizing the different sciences into a single comprehensive view of that fragment of the world that science has succeeded in exploring.

Bertrand Russel was a mathematician and a philosopher. He believed strongly in two possible sources of information, mathematics and science. Why? These two academic areas alone provided empirical knowledge. The other subject matter areas, to Russell, were subjective and lacked reliability. Mathematics is precise and exact with its many patterns and formulas, according to Russell.

Experimentalism emphasizes pupils learning that which is useful and utilitarian. Within a given problem area, mathematics is used to solve selected problems. Committee work is emphasized in that in society, people work in groups to solve problem areas.

Existentialism stresses individual choices made by a pupil in selecting sequential tasks and experiences in mathematics. The pupil is

the chooser. The tasks may involve problem solving as well as other kinds of tasks.

The teacher needs to select that philosophy to implement which assists a pupil to attain optimally. Pupils differ form each other in numerous ways such as native abilities, past experiences, interests, motivation, and purposes. It behooves the mathematics teacher to prepare well and guide learners individually to attain optimally. Use of diverse philosophies of education to provide for individual differences should assist each pupil to learn as much mathematics as possible.

Pertaining to attitude and the affective development of pupils in mathematics, Ediger (1994) wrote:

> In providing for individual differences and to guide optimal affective achievement, the mathematics teacher needs to guide students to achieve an adequate self concept. Adequate self concept development comes about when teachers assist students to:
>
> * Achieve meaningful knowledge so that understating of acquired subject matter is in evidence.
> * Develop readiness for learning in order to have pupils experience sequence in ongoing activities.
> * Increase interest in the mathematics curriculum to attain attention to achieve worthwhile objectives.
> * Perceive purpose in achievement to understand reasons for attaining in ongoing lessons.
> * Enjoy mathematics and thus develop quality attitudes in the affective dimension.

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